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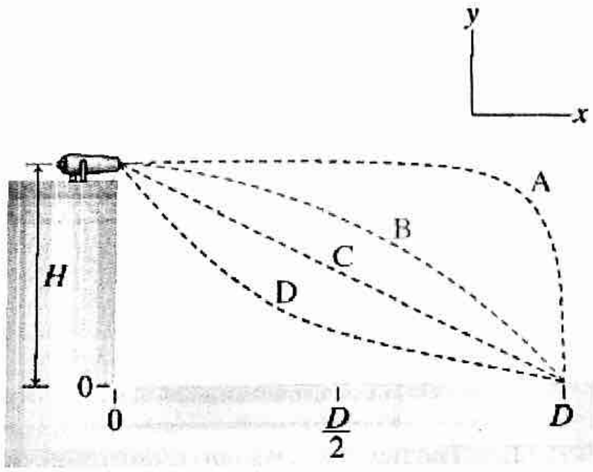
Mastering Physics 2.1 - Projectile Motion

1. A cannon is fired from the top of a cliff as shown in the figure. Ignore drag (air friction) for this question. Take H as the height of the cliff.



   a. Which of the paths would the cannonball most likely follow if the cannon barrel is horizontal?
      - ○ A
      - ◉ B ✓
      - ○ C
      - ○ D

   b. Now the cannon is pointed straight up and fired. (This procedure is not recommended!) Under the conditions already stated (drag is to be ignored) which of the following correctly describes the acceleration of the ball?
      - ○ A steadily increasing downward acceleration from the moment the cannonball leaves the cannon barrel until it reaches its highest point
      - ○ A steadily decreasing upward acceleration from the moment the cannonball leaves the cannon barrel until it reaches its highest point
      - ○ A constant upward acceleration
      - ◉ A constant downward acceleration ✓

2. PhET Simulation

   a. Drag the cannon downwards so it is at ground level, or 0 m (which represents the initial height of the object), then fire the pumpkin straight upward (at an angle of 90°) with an initial speed of 14 m/s. How long does it take for the pumpkin to hit the ground?

      $y_0 = 0$ m $v_f = 0$ m/s $v = v_0 + at$ $t = 1.43 \times 2 =$ **2.86 sec**

      $v_0 = 14$ m/s $0 = 14 + (-9.81)t$

      $a = -9.81$ m/s² $t = 1.428571$ s

   b. When the pumpkin is shot straight upward with an initial speed of 14 m/s, what is the maximum height above its initial location?

      $\Delta y = ?$ $\Delta y = \left(\frac{v + v_0}{2}\right)t$ **$\Delta y = 10$ m**

      $\Delta y = \left(\frac{0+14}{2}\right) 1.43$

   c. If the initial speed of the pumpkin is doubled, how does the maximum height change?
      - ○ The maximum height increases by a factor of two.
      - ◉ The maximum height increases by a factor of four.
      - ○ The maximum height increases by a factor of 1.4 (square root of 2).

      $v = v_0 + at$

      $0 = 28 + (-9.8)t$ $\Delta y = 40$ m.

      $t = 2.857$ sec

   d. Erase all the trajectories, and fire the pumpkin vertically again with an initial speed of 14 m/s. As you found earlier, the maximum height is 9.99 m. If the pumpkin isn't fired vertically, but at an angle less than 90°, it can reach the same maximum height if its initial speed is faster. Set the initial speed to 22 m/s, and find the angle such that the maximum height is roughly the same. Experiment by firing the pumpkin with many different angles. What is this angle?
      - ○ 35°
      - ◉ 40°
      - ○ 45°
      - ○ 50°
      - ○ 55°

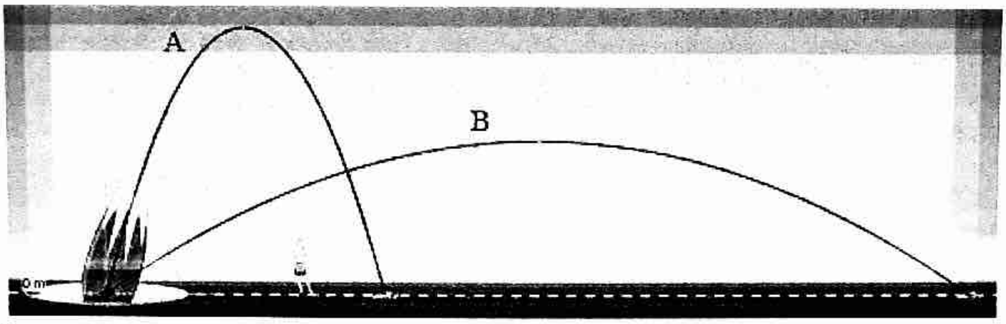
      $\sin\theta \cdot 22 = 14$

      $\sin\theta = \frac{7}{11}$

      $\theta \approx 39.5°$

      $\approx 40°$

e. In the previous part, you found that a pumpkin fired with an initial speed of 22 m/s and an angle of 40° reaches the same height as a pumpkin fired vertically with an initial speed of 14 m/s. Which pumpkin takes longer to land?
   - Both pumpkins are in the air the same amount of time.
   - The pumpkin fired vertically stays in the air longer.
   - The pumpkin fired at an angle of 40° stays in the air longer.



f. The figure shows two trajectories, made by two pumpkins launched with different angles and possibly different initial speeds.Based on the figure, for which trajectory was the pumpkin in the air for the greatest amount of time?
   - It's impossible to tell solely based on the figure.
   - Trajectory B
   - Trajectory A
   - The pumpkins are in the air for the same amount of time.

g. The range is the horizontal distance from the cannon when the pumpkin hits the ground. This distance is given by the product of the horizontal velocity (which is constant) and the amount of time the pumpkin is in the air (which is determined by the vertical component of the initial velocity, as you just discovered). Set the initial speed to 14 m/s, and fire the pumpkin several times while varying the angle between the cannon and the horizontal. For which angle is the range a maximum (with the initial speed held constant)?
   - 0°
   - 45°
   - 90°
   - 30°
   - 60°

h. How does the range of the pumpkin change if its initial velocity is tripled (keeping the angle fixed and less than
   - The pumpkin's range is nine times as far.
   - The pumpkin's range is eighteen times as far.
   - The pumpkin's range is three times as far.

vertical component x 3
velocity.

i. Now, let's see what happens when the cannon is high above the ground. Click on the cannon, and drag it upward as far as it goes (15 m above the ground). Set the initial velocity to 14 m/s, and fire several pumpkins while varying the angle. For what angle is the range the greatest?
   - 45°
   - 40°
   - 50°
   - 30°
   - 20°

j. So far in this tutorial, you have been launching a pumpkin. Let's see what happens to the trajectory if you launch something bigger and heavier, like a car. Compare the trajectory and range of the pumpkin to that of the car, using the same initial speed and angle (e.g., 45°). (Be sure that air resistance is still turned off.) Which statement is true?

- The trajectories differ; the range of the car is shorter than that of the pumpkin.
- The trajectories differ; the range of the car is longer than that of the pumpkin.
- The trajectories and thus the range of the car and the pumpkin are identical.

k. In the previous part, you discovered that the trajectory of an object does not depend on the object's size or mass. But if you have ever seen a parachutist or a feather falling, you know this isn't really true. That is because we have been neglecting air resistance, and we will now study its effects here. For the following parts, select the "Lab" mode of the simulation found at the bottom of the screen. Notice that you can adjust the mass and diameter of the object being launched. Turn on Air Resistance by checking the box. Fire a cannonball with an initial speed of 18 m/s and an angle of 45°. Compare the trajectory to the case without air resistance. How do the trajectories differ?

- The trajectory with air resistance has a shorter range.
- The trajectory with air resistance has a longer range.
- The trajectories are identical.

l. What happens to the trajectory of the cannonball when you increase the diameter while keeping the mass constant?

- Increasing the size makes the range of the trajectory decrease.
- Increasing the size makes the range of the trajectory increase.
- The size of the object doesn't affect the trajectory.

m. You might think that it is never a good approximation to ignore air resistance. However, often it is. Fire the cannonball without air resistance, and then fire it with air resistance (same angle and initial speed). Then, adjust the mass of the cannonball (increase it and decrease it) and see what happens to the trajectory. Don't change the diameter. When does the range with air resistance approach the range without air resistance?

- It never does. Regardless of the mass, the range with air resistance is always shorter than the range without.
- The range with air resistance approaches the range without air resistance as the mass of the cannonball is decreased.
- The range with air resistance approaches the range without air resistance as the mass of the cannonball is increased.

3. The crew of a cargo plane wishes to drop a crate of supplies on a target below. To hit the target, when should the crew drop the crate? Ignore air resistance.

- When the plane is directly over the target
- Before the plane is directly over the target
- After the plane has flown over the target

4. Which projectile spends more time in the air, the one fired from 30° or the one fired from 60°?

- The one fired from 60°
- The one fired from 30°
- They both spend the same amount of time in the air.

5. Consider the video demonstration that you just watched. A more complete explanation of what *you* saw will be possible after covering Newton's laws. For now, consider the following question: How would the result of this experiment change if we replaced the ball with another one that had half the mass? Ignore air resistance.
   - o The ball would land ahead of the cart.
   - o The ball would still land in the cart.
   - o The ball would land behind the cart.

6. Which ball (if either) has the greatest speed at the moment of impact?
   - o Both balls have the same speed.
   - o The ball thrown horizontally
   - o The dropped ball

7. An arrow is shot at an angle of θ=45° above the horizontal. The arrow hits a tree a horizontal distance D=220m away, at the same height above the ground as it was shot. Use g=9.8m/s2 for the magnitude of the acceleration due to gravity.
   a. Find t_a, the time that the arrow spends in the air. Answer numerically in seconds, to two significant figures.
   Solve using only variables, then substitute relevant values.

$\theta = 45°$.
$D = 220m$
$g = 9.8 m/s^2$
$v_0 = 46.43 m/s$.

$v_0 = \sqrt{\frac{Dg}{\sin 2\theta}}$

$v_0 = \sqrt{\frac{(220)(9.8)}{\sin(2(45°))}}$

$v_0 \approx 46.43 m/s$

$t_a = \frac{2 V_0 \sin\theta}{g}$

$t_a = \frac{2(46.43)\sin 45°}{9.8}$

$t_a \approx 6.7 sec$

   b. How long after the arrow was shot should the apple be dropped, in order for the arrow to pierce the apple as the arrow hits the tree?
   Solve using only variables, then substitute relevant values.

$\Delta y = v_0 t + \frac{1}{2} a t^2$

$h = 0 + \frac{1}{2} g t^2$

$t^2 = \frac{2h}{g}$

$t = \sqrt{\frac{2h}{g}}$.

$t_d = \sqrt{\frac{2(6)}{9.8}}$

$t_d = 1.11 s$.

$t = t_a - t_d$.

$= 6.7s - 1.11s$

$\approx 5.6 sec$

8. The archerfish is a type of fish well known for its ability to catch resting insects by spitting a jet of water at them. This spitting ability is enabled by the presence of a groove in the roof of the mouth of the archerfish. The groove forms a long, narrow tube when the fish places its tongue against it and propels drops of water along the tube by compressing its gill covers.
When an archerfish is hunting, its body shape allows it to swim very close to the water surface and look upward without creating a disturbance. The fish can then bring the tip of its mouth close to the surface and shoot the drops of water at the insects resting on overhead vegetation or floating on the water surface.

   a. At what speed v should an archerfish spit the water to shoot down an insect floating on the water surface located at a distance 0.800 m from the fish? Assume that the fish is located very close to the surface of the pond and spits the water at an angle 60° above the water surface.
   Solve using only variables, then substitute relevant values.

$\Delta x = D = 0.800\text{m}$

$V_0 = ?$

$\theta = 60°$

$t = \frac{2v_0 \sin\theta}{g}$

$t = \frac{2v_0 \sin 60°}{9.8} = 0.1767 v_0$

$\Delta x = v_0 t \cos\theta$

$0.800 = v_0 \cdot 0.1767 v_0 \cdot \cos 60°$

$0.1767 v_0^2 = 1.6$

$\boxed{v_0 \approx 3.01 \text{ m/s}}$

   b. Now assume that the insect, instead of floating on the surface, is resting on a leaf above the water surface at a horizontal distance 0.600 m away from the fish. The archerfish successfully shoots down the resting insect by spitting water drops at the same angle 60° above the surface and with the same initial speed v as before. At what height h above the surface was the insect?
   Solve using only variables, then substitute relevant values.

$y = v \cos\theta \, t$

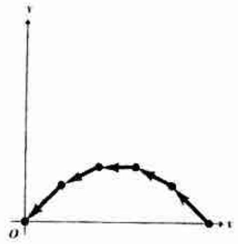
$t = \frac{y}{v\cos\theta} = \frac{0.6\text{m}}{(3.01)\cos 60°} = 0.3987 \text{ sec}$

$h = v \sin\theta \, t - \frac{1}{2} g t^2$

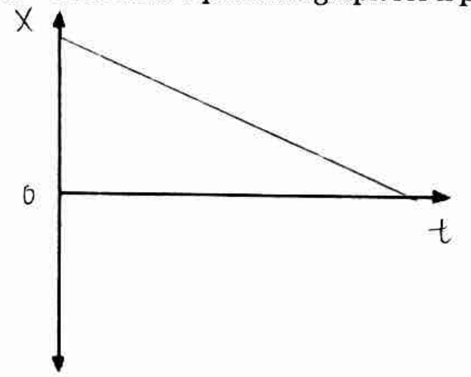
$h = (3.01)\sin 60° \cdot 0.3987 - \frac{1}{2}(9.8)(0.3987)^2$

$\boxed{h \approx 0.260 \text{ m}}$

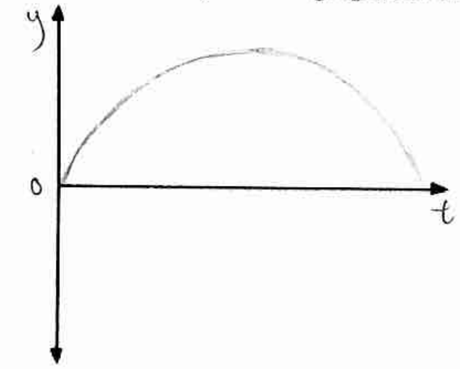
9. For the motion diagram given (Figure 1), sketch the shape of the corresponding motion graphs in Parts A to D. Use the indicated coordinate system. One unit of time elapses between consecutive dots in the motion diagram.



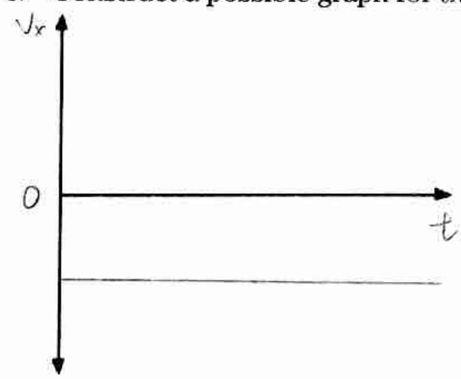
a. Construct a possible graph for x position versus time, $x(t)$.



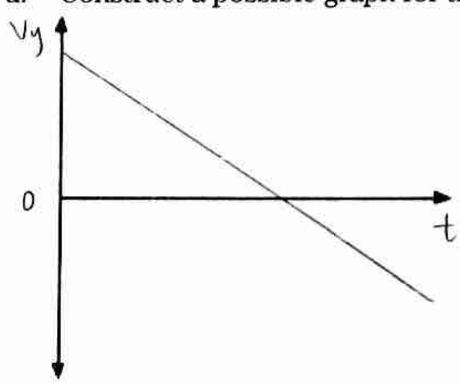
b. Construct a possible graph for the y position versus time, $y(t)$.



c. Construct a possible graph for the x velocity versus time, $v_x(t)$.

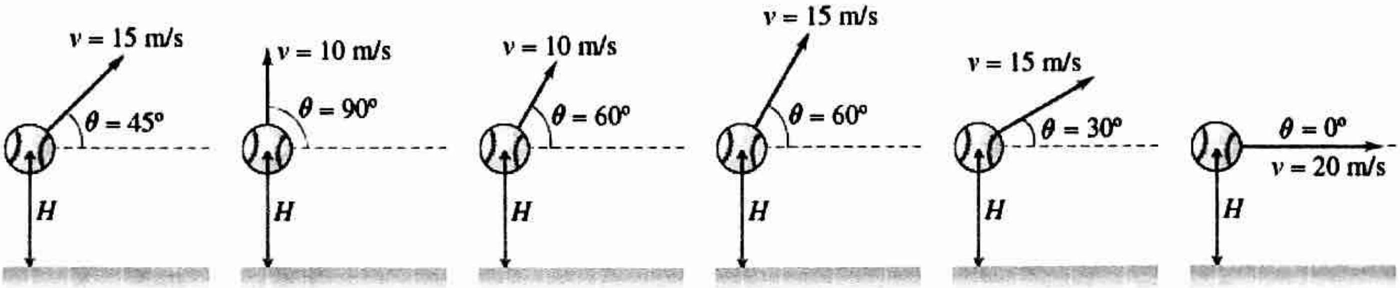


d. Construct a possible graph for the y velocity versus time, $v_y(t)$.



10.

a. Rank these throws based on the maximum height reached by the ball.
Rank from largest to smallest. To rank items as equivalent, overlap them.



$$H = \frac{v^2 \sin^2 \theta}{2g}$$

$H_1 = \frac{(15)^2 \sin^2 45°}{2(9.8)} = 5.74 m$

$H_2 = \frac{(10)^2 \sin^2 90°}{2(9.8)} = 5.1 m$

$H_3 = \frac{(10)^2 \sin^2 60°}{2(9.8)} = 3.83 m$

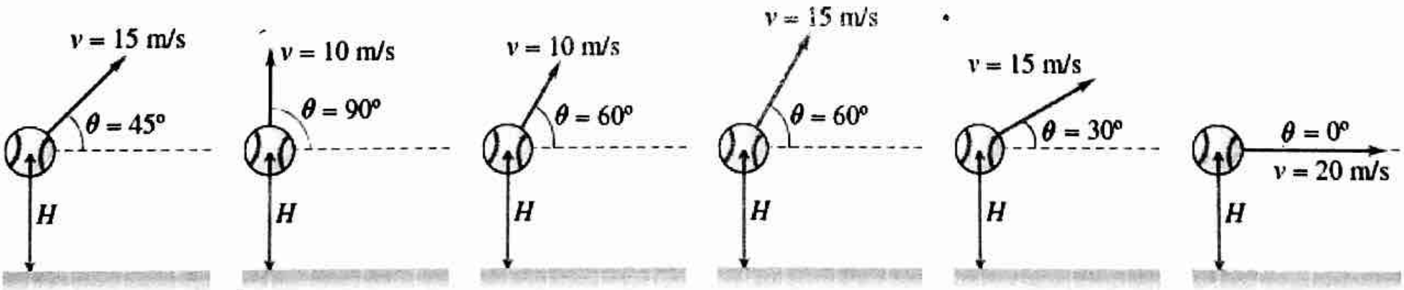
$H_4 = \frac{(15)^2 \sin^2 60°}{2(9.8)} = 8.61 m$

$H_5 = \frac{(15)^2 \sin^2 30°}{2(9.8)} = 2.87 m$

$H_6 = \frac{(20)^2 \sin^2 0°}{2(9.8)} = 0 m$

$H_4 > H_1 > H_2 > H_3 > H_5 > H_i$

b. Rank these throws based on the amount of time it takes the ball to hit the ground.
Rank from largest to smallest. To rank items as equivalent, overlap them.



$$t = \frac{2 v_0 \sin \theta}{g}$$

$t_1 = \frac{2(15) \sin 45°}{9.8} = 2.16 sec$

$t_2 = \frac{2(10) \sin 90°}{9.8} = 2.04 sec$

$t_3 = \frac{2(10) \sin 60°}{9.8} = 1.767 sec$

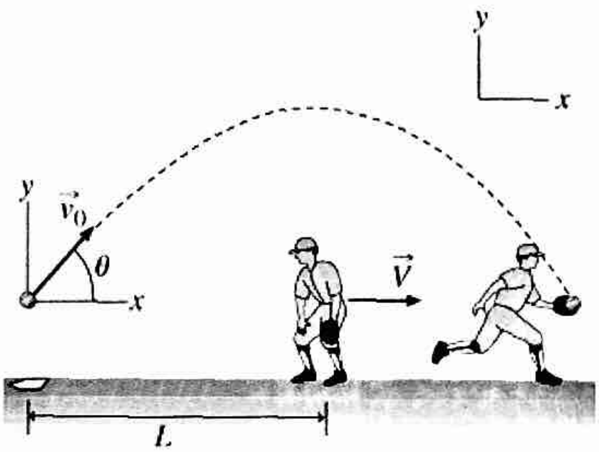
$t_4 = \frac{2(15) \sin 60°}{9.8} = 2.65 sec$

$t_5 = \frac{2(15) \sin 30°}{9.8} = 1.53 sec$

$t_6 = \frac{2(20) \sin 0°}{9.8} = 0 sec$

$t_4 > t_1 > t_2 > t_3 > t_5 > t_0$

11. (Figure 1) A softball is hit over a third baseman's head with some speed v0 at an angle θ above the horizontal. Immediately after the ball is hit, the third baseman turns around and begins to run at a constant velocity V=7.00m/s. He catches the ball t=2.00s later at the same height at which it left the bat. The third baseman was originally standing L=18.0m from the location at which the ball was hit.



a. Find v_0. Use $g=9.81m/s^2$ for the magnitude of the acceleration due to gravity.
Solve using only variables, then substitute relevant values.

$\Delta x = v t = (7\,m/s)(2\,s) = 14m$

$\Delta x = D_{total} = 18 + 14 = 32\,m$

$t = 2s$

$V_{0x} = \frac{d}{t} = \frac{32m}{2} = 16m/s$

$V_f = V_{0y} - gt$

$V_{0y} = (9.8m/s^2)(1s) = 9.8m/s$

$V_0 = \sqrt{V_{0x}^2 + V_{0y}^2} = \sqrt{16^2 + 9.8^2} = \boxed{18.8\,m/s}$

b. Find the angle θ in degrees.
Solve using only variables, then substitute relevant values.

$D = \frac{V_0^2 \sin 2\theta}{g}$

$32m = \frac{(18.8m/s)^2 \sin 2\theta}{9.8}$

$18.8^2 \sin 2\theta = 313.6$

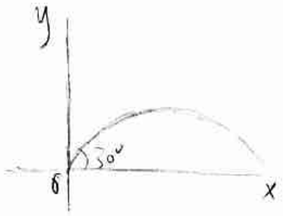
$\sin 2\theta = 0.8873$

$2\theta = 63$

$\boxed{\theta \approx 31.5°}$

12. A frog jumps at an angle 30° above the horizontal. The origin of the coordinate system is at the point where the frog leaves the ground. Assume the x-axis is directed horizontally in the direction of the frog's motion, and the y-axis is directed upward.

Complete the table by putting cross and check marks in the cells. If a physical quantity in the first column that describes the motion of the frog corresponds to the description in the first row of what is happening to this quantitiy while the frog is moving, put a check mark into the cell, but otherwise put a cross mark. Consider the frog as a point-like object, and assume that the resistive force exerted by the air is negligible.



Physical quantity	Remains constant	Is changing	Increases only	Decreases only	Increases, then decreases	Decreases, then increases
x-coordinate magnitude	✗	✓	✓	✗	✗	✗
y-coordinate magnitude	✗	✓	✗	✗	✓	✗
Direction of velocity	✗	✓	✗	✓	✗	✗
Magnitude of velocity	✗	✓	✗	✗	✗	✓
Direction of acceleration	✓	✗	✗	✗	✗	✗
Magnitude of acceleration	✓	✗	✗	✗	✗	✗

13. You can shoot an arrow straight up so that it reaches the top of a 33 -m-tall building. The arrow starts 1.45 m above the ground.

   a. How far will the arrow travel if you shoot it horizontally while pulling the bow in the same way? Neglect the air resistance.
   Solve using only variables, then substitute relevant values.

$h = 33 - 1.45 = 31.55\ m$

$h = \frac{v^2}{2g}$

$31.55 = \frac{v^2}{2(9.8)}$

$v^2 = 618.38$

$v = 24.87\ m/s$

$t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{2(1.45)}{9.8}} = 0.544\ sec.$

$\Delta x = vt = 0.544 \times 24.87 \approx 13.5\ m$

   b. Where do you need to put a target that is 1.45 m above the ground in order to hit it if you aim 30 ° above the horizontal while pulling the bow in the same way?
   Solve using only variables, then substitute relevant values.

$y = v_{0y}t + \frac{1}{2}gt^2$ $\qquad y = 1.45 - 1.45 = 0\ m$

$0 = (24.87)(\cos 30°)(\sin 30°)t - \frac{1}{2}gt^2$

$t = \frac{2(24.87 \cos 30° \sin 30°}{9.8}$

$t = 2.198\ sec.$

$x = v_0 t = (24.87 \cos 30°)(2.198\ sec) \approx 54.65\ m$